SHRI RAMSWAROOP MEMORIAL UNIVERSITY

End Semester Examination (2021-22)-Odd Semester

M. Sc. (Mathematics) I Year (I Sem)	
Course Name: Real Analysis	**Code: MMA1002**
Time: 0 Hours	**Max Marks: 60**

University Roll No.															
(To be filled by the Student)															

Note: Please read instructions carefully:

a) *The question paper has 03 sections and it is compulsory to attempt all sections.*

b) *All questions of Section A are compulsory; questions in Section B and C contain choice.*

	Section A: Very Short Answer type Questions **Attempt all the questions.**	**BL**	**CLO**	**Marks (10)**
1.	Find the limit points of the sequence $< 2+(-1)^n >$.	BL2	CLO4	02
2.	Define "Pseudo metric space".	BL1	CLO2	02
3.	Show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$, $p>1$ is uniformly convergent for real values of x.	BL3	CLO1	02
4.	Prove that every compact subset of a metric space is closed.	BL5	CLO2	02
5.	Calculate $L(P,f)$ and $U(P,f)$ if $f(x)=x^3$ on [0, 1] and $P=\{0,1/4,2/4,3/4,1\}$ be a partition of [0, 1].	BL3	CLO3	02
	Section B: Short Answer Type Questions **Attempt any 03 out of 05 questions.**	**BL**	**CLO**	**Marks (30)**
1.	Examine the following function for continuity and differentiability at $x=0$ and $x=1$. $$y=\begin{cases} x^2, \ for\ x\le 0 \\ 1, \ for\ 0<x\le 1 \\ 1/x, \ for\ x>1 \end{cases}$$	BL3	CLO4	10
2.	Show that the series $\sum \frac{(-1)^{n-1}}{n+x^2}$ is uniformly convergent for all value x.	BL3	CLO1	10
3.	Let R be the set of real number. Show that the function $d: R\times R \to R$ defined by $d(x,y)=\lvert x-y\rvert \ \forall\ x,y\in R$ is a metric on R.	BL3	CLO2	10
4.	If A and B are two subsets of a metric space (X,d) then prove that $\overline{(A\cap B)} \subseteq \overline{A}\cap\overline{B}$.	BL5	CLO2	10
5.	Prove that if P_1 and P_2 be any two partition of $[a,b]$, then $U(P_2,f,\alpha)\ge L(P_1,f,\alpha)$. that is no lower sum can exceed any upper	BL5	CLO3	10

		BL	CLO	Marks
	sum.			
	Section C: Long Answer Type Questions **Attempt any 01 out of 04 questions.**	**BL**	**CLO**	**Marks (20)**
1.	State & prove Cauchy's second theorem on limits and hence show that $\lim_{n\to\infty}\left[\frac{2}{1}\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\cdots\left(\frac{n+1}{n}\right)^n\right]^{1/n}=e$ where $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e.$	BL3	CLO4	20
2.	Show that the series $\sum_{n=1}^{\infty}\frac{x}{(nx+1)\{(n-1)x+1\}}$ is uniformly convergent on any interval [a, b] where $0<a<b,$ but only point wise convergent on [0, b].	BL5	CLO1	20
3.	Let (X,d_1) and (Y,d_2) be two metric spaces. Prove that a function $f:X\to Y$ is continuous on X if and only if for each open set $G\subset Y,\ f^{-1}(G)$ is an open subset of X.	BL5	CLO2	20
4.	If f is monotonic on [a, b], and if α is continuous and monotonic increasing on [a, b], then show that f is RS- integrable.	BL3	CLO3	20
